Pergamon

Int. J. Multiphase Flow Vol. 24, No. 2, pp. 335–345, 1998

Printed in Great Britain
0301-9322/98 $19.00 + 0.00

PII: S0301-9322(97)00066-9

IMPROVED EDDY INTERACTION MODELS WITH RANDOM LENGTH AND TIME SCALES

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(Received 28 August 1996; in revised form 1 September 1997)

Abstract—The eddy interaction model has been used extensively to model particle dispersion in complex turbulent flows. In the model, a particle undergoes a series of interactions with random-velocity eddies. Interaction times, which determine particle dispersion are influenced by the eddy velocity and eddy length and time scales. In general, these can all be random. Recent research has shown some of the shortcomings of the original model, and has suggested improvements be made to ensure that models account for the crossing trajectories, inertia and continuity effects. In this present paper, the performance of variants of the improved model in predicting dispersion of particles in a simple turbulent flow one investigated. Each variant is given a different combination of eddy length and time distributions. Numerical results are compared with previously published analytical results. It is demonstrated that the effects noted above are allowed for in each of the model combinations considered.

Key Words: particle dispersion, turbulent flow, Lagrangian models, integral scales

1. INTRODUCTION

The eddy interaction model (EIM) developed by Gosman and Ioannides (1981) is one of the simplest and most frequently-used methods for simulation of turbulent particle dispersion. In the EIM, individual particles undergo a series of interactions with random-velocity fluid eddies. A particle interacts with an individual eddy so long as the particle remains within that eddy and during each interaction the eddy velocity remains constant. In the original model, the particle remains within the eddy until either the eddy "dies" when the "eddy lifetime" t_e is exceeded, or the particle "crosses" the eddy, for example, when the separation between the particle and the centre of the eddy exceeds the eddy length l_e. Particle motions are determined by evaluating the influence of viscous drag and other forces over the duration of the interaction. On exit from an eddy, the particle immediately enters another eddy with generally different characteristics. Eventually, particle phase data are determined by statistical averaging over a large number of trajectories.

For particle dispersion in homogeneous isotropic and stationary turbulence (HIST), the original model of Gosman and Ioannides (1981) would give eddy length and time scales which do not change from eddy to eddy. More recently, however, randomly-sampled scales have been used. Kallio and Reeks (1989) used time scales sampled from an exponential probability distribution, while Burnage and Moon (1990) used exponential distributions for both time and length scales. Wang and Stock (1992) used several different time scale distributions and developed a general method to find the Lagrangian integral time scale τ_L for a given distribution. In this paper, we investigate the performance of four different eddy interaction models with random length and times scales.

We note here that, traditionally, t_e has been called the "eddy lifetime". Following Graham (1996b), however, and to avoid confusion later, we call t_e the fluid particle interaction time or FPIT. Wang and Stock (1992) showed that the probability distribution chosen for the FPIT determines the Lagrangian fluid velocity auto-correlation. Graham and

James (1996) extended Wang and Stock's method to show that the distribution of the eddy length l_e determines an Eulerian spatial velocity correlation. By allowing random time and length scales in eddy interaction models, it is therefore possible to describe real turbulent flows more accurately, by specifying particular correlation functions or spectral behaviour. Particular distributions of eddy characteristics might be chosen in order to match experimentally-determined forms for these auto-correlations or, equivalently, to match particular frequency or wave-number spectra.

Because the FPIT distribution determines the Lagrangian auto-correlation, it also fixes the Lagrangian integral time scale τ_L. Similarly, the distribution of l_e fixes the Eulerian longitudinal length scale Λ_E. Recall that the diffusion coefficient of fluid particles in HIST is $u'^2\tau_L$ and the dispersion of heavy particles settling at a velocity v_g under gravity is $u'^2\Lambda_E/v_g$. By ensuring the correct integral time and length scales, the original model is therefore capable of accounting for Yudine's (Yudine 1959), crossing trajectories effect (CTE).

The original model uses the constraint that eddy/particle interaction times can never exceed the corresponding interaction times for fluid particles. Graham and James showed that, because of this constraint, finite-inertia particles disperse less rapidly in the long term limit than fluid particles. This was shown to be the case whatever the choice of eddy length and time scales. This property of "standard" eddy interaction models is contrary to analytical and experimental results which show that dispersivity can increase with particle inertia, a phenomenon called the inertia effect (Reeks 1977, Wells and Stock 1983, Deutsch 1992, Squires and Eaton 1991).

In order to avoid this problem, Graham (1996a) proposed a modified EIM which has two different time scales. In the modified model, finite-intertia particles are allowed to interact with eddies for a maximum interaction time $t_{\max}$, which may be greater than the interaction time for fluid particles (i.e. the FPIT). Graham (1996a) specified a constant maximum interaction time $t_{\max} = T_{\max}$, whereas in this present paper, $t_{\max}$ can be randomly-sampled. This method allows for the possibility that the (moving) Eulerian integral time scale τ_E can be greater than the Lagrangian integral time scale τ_L. In this present paper, the method developed in Graham (1996a) for the case of constant FPIT and constant eddy length is extended to three other combinations of FPIT and eddy length distributions. This ensures that any of the the model combinations can predict the enhanced dispersion of high-inertia (but low drift velocity) particles.

The original EIM also fails to model the "continuity effect" Csanady (1963), whereby particle dispersion at right angles to a strong drift velocity is less than the dispersion parallel to the drift. The effect is due to the difference between "lateral" and "longitudinal" length scales, which is in turn due to the continuity of the fluid turbulence. This failure of the original EIM to predict this effect is because it specifies that interaction times are identical in all coordinate directions, leading to identical dispersivity in all directions. Graham (1996b) developed a method to allow for this "continuity effect" when the FPIT and eddy length are constant. The method is also used below with the other combinations of FPIT and eddy length.

Other Lagrangian models are of course available which account for the CTE, inertia and continuity effects. The contributions of Pozorski *et al.* (1993), Lu *et al.* (1993), Huang *et al.* (1993) and Wang and Stock (1994), for example, include these effects. Pozorski *et al.* (1993) and Lu *et al.* (1993) both used methods based upon the Langevin equation (Kloeden and Platen 1995), while Wang and Stock (1994) used Kraichnan's (Kraichnan 1970) method of random Fourier modes. More closely related to the present paper is the eddy interaction method developed by Huang *et al.* (1993).

There are two main differences between Huang's method and the method used in this paper. Firstly, Huang uses a maximum interaction time which depends on particle intertia. This determines the relationship between dispersion coefficients and particle inertial time scale τ_p when body forces are absent. It should be noted that Huang's relationship is strictly valid for only one value of the "turbulence structure parameter" $\beta = u'\tau_L/\Lambda_E$ (where u' is the turbulence intensity). The method used below is similar to that used in Graham (1996a,b) in which $T_{\max}$ is independent of the particle properties but depends on β. The resulting method is then valid for any value of β. The relationship between dispersion coefficient and particle inertia is then dependent on the distributions of eddy velocity, length and time scales and on β. The main feature is that the method is designed to perform properly in the limiting cases where $\tau_p \to 0$, $\tau_p \to \infty$, whatever the value of β.

The second main difference between Huang's method and the present is the expression used to evaluate the "crossing time". Huang used a simple expression to evaluate these times, whereas Graham (1996b) used a more complex and costly expression which involved an iterative method to include the effects of gravity. However, it can be shown that Huang's method under-predicts dispersion of fluid particles, especially if the turbulence structure parameter is not small. Graham ensures that fluid particle diffusion is correctly predicted, whatever the value of β. The method described below uses a method which combines the economy of the method of Huang *et al.*, with Graham's method of ensuring the correct diffusivity of fluid particles. In effect both Huang *et al.* and Graham used constant eddy length and time scales (that is. these scales were not randomly-sampled). In this paper, the performance of eddy interaction models with *random* eddy scales is evaluated. In addition to the usual random eddy velocity, then, the FPIT t_e, the maximum interaction time $t_{\max}$ and the eddy length l_e are also random. The aim is to account for the CTE, inertia and continuity effects in these variants.

The organisation of the remainder of this paper is as follows. In Section 2, the modified eddy interaction model is described. Four different combinations of eddy length and time scale distributions used are described and each model is designed to have the same τ_L and Λ_E. Eulerian time scales Λ_E are discussed in Section 3 and are shown to depend on the eddy length and time scale distributions. The relationship between Λ_E and the EIM parameters is therefore shown to be different for each model combination. In Section 4, the results obtained using each model are compared with the analytical results of Reeks (1977). Consistency of integral length and time scales is ensured prior to the computations and it is shown that the results from each model agree well with the analytical expressions.

2. MODEL SPECIFICATIONS

The purpose of this paper is to investigate the performance of different versions of the eddy interaction model in modelling particle dispersion in turbulent flows. The performance of the different models is compared with the analytical solution of Reeks (1977). Reeks' analysis required several simplifying assumptions, including the following (i) the underlying turbulence is homogeneous, isotropic and statistically stationary, (ii) particles are spherical and small compared with turbulence scales, (iii) the viscous drag on a particle is given by Stokes' law, and (iv) lift/spin forces are negligible. In order to compare with Reeks' results, the same assumptions are made here.

The motion of a high-density spherical particle under the influence of viscous drag and gravitational forces is determined by

$$\frac{\mathrm{d}\mathbf{u}_p}{\mathrm{d}t} = \frac{(\mathbf{u}_f - \mathbf{u}_p)}{\tau_p} + \mathbf{g}, \quad [1]$$

where $\boldsymbol{u}_p$ is the particle velocity and $\boldsymbol{u}_f$ is the undisturbed instantaneous fluid velocity at the particle location, τ_p is the particle relaxation time and $\boldsymbol{g}$ is the acceleration due to gravity. In the case of the Stokesian drag, where the particle Reynolds number is small, τ_p is a constant. All quantities are understood to be dimensionless, having been normalised with respect to following characteristic scales: time = τ_L, velocity = u', length = $u'\tau_L$ and acceleration = u'/τ_L.

In the EIM, the fluid velocity is constant during each interaction and [1] can be integrated analytically to provide updated velocities and positions as a result of each interaction. In order to complete the integrations, the interaction time t_i during which a particle is located within an eddy must be found.

For each of the eddy interaction model combinations considered in this paper, the following scheme is used to determine interaction times:

if $(v_g \le \Lambda_E/\tau_L)$ then

$$t_{i_1} = t_{i_2} = t_{i_3} = \begin{cases} t_e, & \text{if } (u_r\tau_p \le l_e); \\ \min[t_{\max}, -\tau_p \log_e(1 - l_e/(u_r\tau_p))], & \text{otherwise;} \end{cases}$$

if $(v_g > \Lambda_E/\tau_L)$ then

$$t_{i_1} = \min(t_{\max},\ l_e/u_r)$$

$$t_{i_2} = t_{i_3} = \begin{cases} t_{i_1}, & \text{if (le/urmax);} \\ t_{i_1}/2, & \text{otherwise.} \end{cases}$$

In the above scheme, t_{i_j} represents the interaction time in the x_j-direction, u_r represents the magnitude of the relative fluid/velocity at the start of an interaction and gravity acts in the x_1 direction. Lower-case variables t_e, $t_{\max}$ and l_e are *instantaneous* values of the FPIT, maximum interaction time and eddy length, randomly sampled and valid for a single eddy. The possibility of having different interaction times in directions parallel to gravity (longitudinal) and at right angles to gravity (lateral) is the modification to the standard model which allows for the continuity effect (see Graham 1996b).

The interaction time used in this present paper is determined using a method similar to that used in Graham (1996b). The expression used for the crossing time depends on the relationship between the drift velocity $v_g = g\tau_p$ and a characteristic velocity scale Λ_E/τ_L. If $v_g < \Lambda_E/\tau_L$, the same expression as used by Graham (1996b) is used, which again neglects the gravitational influence. In this expression, particles can be "trapped" if the stopping distance $u_r\tau_p$ is less than the eddy length. Suitable choice of the distribution of t_e ensures the correct diffusivity of fluid particles, which are always trapped. Suitable choice of the distribution $t_{\max}$ also leads in this case to the correct dispersivity of high-inertia particles when gravity is negligible.

Graham (1996b) also showed that the use of this expression when $v_g > \Lambda_E/\tau_L$ can lead to overpredictions of particle dispersivity when the drift velocity is of the order of the turbulence intensity. Graham then used an iterative strategy to determine interaction times when the influence of gravity on the crossing time was accounted for. Here, however, we note instead that, if $v_g > \Lambda_E/\tau_L$, drift is dominant and the particle velocity is relatively unchanged as it crosses an eddy. In this case, we use the expression of Huang *et al.* (1993) which estimates the interaction time as l_e/u_r, where l_e is the eddy length and u_r is the relative velocity at the beginning of the interaction. We therefore avoid using the iterative strategy of Graham (1996b) and the influence of gravity on the crossing time is not explicitly accounted for. As can be seen in the results section, Huang's expression leads to good predictions and has the advantage of being computationally inexpensive compared with the iterative method.

The interaction time is influenced by the FPIT (t_e), maximum interaction time ($t_{\max}$) and eddy length (l_e), as well as the relative velocity (u_r). The performance of eddy interaction models should therefore be expected to be influenced by different specifications for t_e, $t_{\max}$ and l_e. This extent of this influence is investigated here. Each of the four different models considered in this paper has either (a) constant FPIT and maximum interaction time, or (b) FPIT and maximum interaction time randomly sampled from an exponential probability distribution. Similarly, each of the four models has either (i) constant eddy length, or (ii) eddy length sampled from an exponential probability distribution. In principle, it would be possible to have combinations with constant FPIT but exponentially-distributed $T_{\max}$, or vice versa. For the models considered here, the maximum interaction time follows a similar distribution to that of the FPIT. This ensures that the models to revert to the corresponding "standard models" when the maximum interaction time is set equal to the FPIT.

2.1. *Length distributions*

For the constant eddy-length model, the pdf of the eddy length is given by $g(l_e) = \delta(l_e - 2\Lambda_E)$, where $\delta(l_e)$ is Dirac's delta function. When the eddy length is distributed exponentially, the pdf is $g(l_e) = (1/\Lambda_E)e^{-le/\Lambda_E}$, if $l_e > 0$ and is zero otherwise. We recall the expression derived by Graham and James (1996):

$$\Lambda_E = \frac{\int_0^\infty \int_x^\infty \int_y^\infty g(y)\,\mathrm{d}y\,\mathrm{d}x\,\mathrm{d}l}{\int_0^\infty \int_y^\infty g(y)\,\mathrm{d}y\,\mathrm{d}x} \qquad [2]$$

Both of the model specifications therefore lead to integral length scales in the model equal to Λ_E.

2.2. *Time distributions*

For the constant-FPIT model, the pdf of the FPIT can be written as $f(t_e) = \delta(t_e - 2\tau_L)$. The pdf of the maximum interaction time for this model is $f_1(t_{\max}) = \delta(t_{\max} - T_{\max})$. For the exponentially-distributed FPIT, the pdf is given by $f(t_e) = (1/\tau_L)e^{-t_e/\tau_L}$ if $t_e > 0$ and zero otherwise. The pdf of the maximum interaction time for this model is thus $f_1(t_{\max}) = (1/T_{\max})e^{-t_{\max}/T_{\max}}$ if $t_{\max} > 0$, and zero otherwise.

Wang and Stock's (Wang and Stock, 1992) analysis gives:

$$\tau_L = \frac{\int_0^\infty \int_\tau^\infty \int_t^\infty f(t')\,\mathrm{d}t'\,\mathrm{d}t\,\mathrm{d}\tau}{\int_0^\infty \int_t^\infty f(t')\,\mathrm{d}t'\,\mathrm{d}t} \qquad [3]$$

Both of the FPIT specifications therefore lead to Lagrangian integral time-scales in the model equal to τ_L. Eulerian integral time-scales are dependent upon the distributions of $t_{\max}$, l_e and u_f. Analysis is given below.

3. EULERIAN INTEGRAL TIME-SCALES

Eulerian time-scales are determined by investigating the EIM for the case of fixed-points (or, equivalently, particles with infinite inertia). Clearly, such particles are never "trapped" (which means, interestingly, that τ_E is determined purely by the distribution of $t_{\max}$ and is independent of the FPIT distribution). The time taken for the particle to cross an eddy (that is the time taken for the eddy to sweep over the fixed point) is

$$t_c = l_e/u_f, \qquad [4]$$

where l_e is the eddy length and u_f is the magnitude of the fluid velocity in the eddy [$-\tau_p \log_e(1 - l_e/(u_r\tau_p)) \rightarrow l_e/u_r$ as $\tau_p \rightarrow \infty$ and the particle itself is stationary so that $u_r = u_f$!]. In this case, then the interaction time is always determined by

$$t_i = \min(t_{\max},\ t_c) \qquad [5]$$

Using the analysis of Graham and James (1996), it can be shown that the (moving) Eulerian Integral time-scale is given by

$$\tau_E = \frac{\int_0^\infty \int_\tau^\infty \int_t^\infty f_1(t') \int_0^\infty g(l_e) \int_o^{l_e/t} h(u_f)\,\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t'\,\mathrm{d}t\,\mathrm{d}\tau}{\int_0^\infty \int_t^\infty f_1(t') \int_0^\infty g(l_e) \int_0^{l_e/t} h(u_f)\,\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t'\,\mathrm{d}t}, \qquad [6]$$

where $h(u_f)$ is the pdf of the fluid velocity (here assumed to be normally distributed with mean zero and standard deviation equal to the turbulence intensity u'). The explanation of this expression is as follows:

$$\int_0^{l_e/t} h(u_f)\,\mathrm{d}u_f \qquad [7]$$

represents the probability that the crossing time $t_c = l_e/u_f$ exceeds t, given that the eddy length is l_e. The integral

$$\int_0^\infty g(l_e) \int_0^{l_e/t} h(u_f)\,\mathrm{d}u_f\,\mathrm{d}l_e \qquad [8]$$

therefore represents the total probability that the crossing time is greater than t. The term

$$\int_t^\infty f_1(t')\,\mathrm{d}t' \qquad [9]$$

represents the probability that the maximum interaction time is greater than t. For the case of fixed points, as noted above, the interaction time is equal to the minimum of t_c and $t_{\max}$. The product

$$\phi_E(t) = \int_t^\infty f_1(t')\,\mathrm{d}t' \int_0^\infty g(l_e) \int_0^{l_e/t} 2h(u_f)\,\mathrm{d}u_f\,\mathrm{d}l_e \qquad [10]$$

is therefore the probability that the interaction time exceeds t (assuming that $h(u_f)$ is symmetrical about zero). Graham and James (1996) showed that the auto-correlation of the fluid motion following a particle is given by

$$R_f^p(\tau) = \frac{\int_\tau^\infty \phi(t)\,\mathrm{d}t}{\int_0^\infty \phi(t)\,\mathrm{d}t}, \qquad [11]$$

where $\phi(t)$ is the probability that the interaction time exceeds t. The integral time-scale following the particle is then

$$\tau_f^p = \int_0^\infty R_f^p(\tau)\,\mathrm{d}\tau. \qquad [12]$$

In the case of a fixed-point, then, $\phi = \phi_E$ and $\tau_f^p = \tau_E$, the Eulerian integral time scale, so that

$$\tau_E = \frac{\int_0^\infty \int_\tau^\infty \phi_E(t)\,\mathrm{d}t\,\mathrm{d}\tau}{\int_0^\infty \phi_E(t)\,\mathrm{d}t}, \qquad [13]$$

which is identical to expression [6] on substituting in the expression for $\phi_E(t)$ given in [10]. For a given model, τ_E is therefore dependent on the parameters Λ_E and $T_{\max}$. Specifically, for model (a)(i),

$$\tau_E = \frac{\displaystyle\int_0^{T_{\max}} \int_\tau^{T_{\max}} \int_0^{2\Lambda_E/T_{\max}} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}t\,\mathrm{d}\tau}{\displaystyle\int_0^{T_{\max}} \int_0^{2\Lambda_E/T_{\max}} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}t}, \qquad [14]$$

for model (a)(ii),

$$\tau_E = \frac{\displaystyle\int_0^{T_{\max}} \int_\tau^{T_{\max}} \int_0^\infty \frac{1}{\Lambda_E} e^{-l_e/\Lambda_E} \int_0^{l_e/T_{\max}} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t\,\mathrm{d}\tau}{\displaystyle\int_0^{T_{\max}} \int_0^\infty \frac{1}{\Lambda_E} e^{-l_e/\Lambda_E} \int_0^{l_e/T_{\max}} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t}. \qquad [15]$$

for model (b)(i),

$$\tau_E = \frac{\displaystyle\int_0^\infty \int_\tau^\infty \int_t^\infty \frac{1}{T_{\max}} e^{-t'/T_{\max}} \int_0^{2\Lambda_E/t} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}t'\,\mathrm{d}t\,\mathrm{d}\tau}{\displaystyle\int_0^\infty \int_t^\infty \frac{1}{T_{\max}} e^{-t'/\max} \int_0^{2\Lambda_E/t} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}t'\,\mathrm{d}t}. \qquad [16]$$

for model (b)(ii),

$$\tau_E = \frac{\displaystyle\int_0^\infty \int_\tau^\infty \int_t^\infty \frac{1}{T_{\max}} e^{-t'/T_{\max}} \int_0^\infty \frac{1}{\Lambda_E} e^{-l_e/\Lambda_E} \int_0^{l_e/t} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t'\,\mathrm{d}t\,\mathrm{d}\tau}{\displaystyle\int_0^\infty \int_t^\infty \frac{1}{T_{\max}} e^{-t'/T_{\max}} \int_0^\infty \frac{1}{\Lambda_E} e^{-l_e/\Lambda_E} \int_0^{l_e/t} \frac{1}{\sqrt{2\pi}} e^{-u_f^2/2}\,\mathrm{d}u_f\,\mathrm{d}l_e\,\mathrm{d}t'\,\mathrm{d}t}. \qquad [17]$$

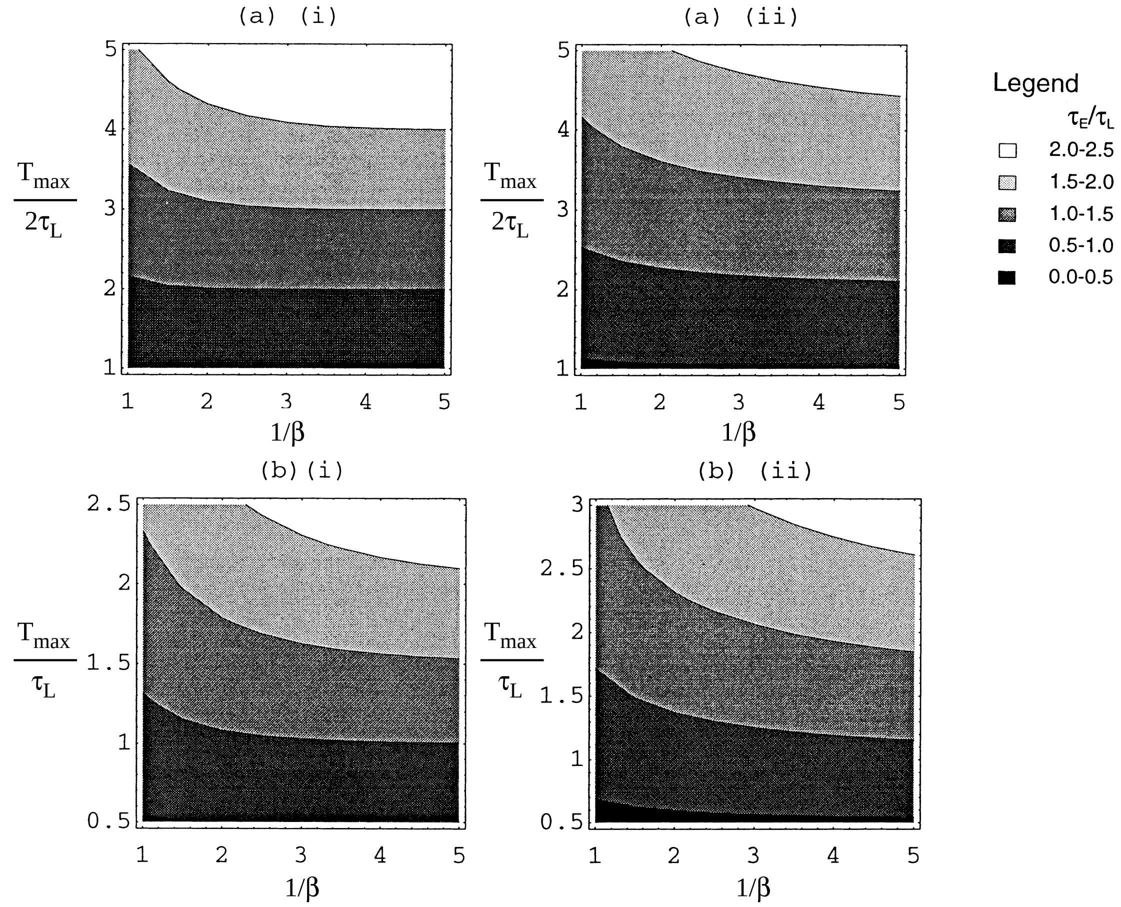

Figure 1. Contours of Eulerian integral time-scale for all model combinations.

We recall that all quantities have been non-dimensionalised in the above expressions. In particular, we note that the dimensionless Eulerian integral length scale $\Lambda_E/(u'\tau_L)$ is the reciprocal of the turbulence structure parameter. In the non-dimensional units, the ratio τ_E/τ_L therefore depends on the turbulence structure parameter β and the ratio $T_{\max}/\tau_L$. Each of the four different schemes considered here leads to a different $\tau_E/\tau_L(\beta, T_{\max}/\tau_L)$ relationship. The relationships have been evaluated by numerical integration using Mathematica (Wolfram 1991) and are illustrated in the form of contour plots in figure 1.

For the constant $t_{\max}$ model (a), the Eulerian integral time scale might be expected to be half of $T_{\max}$, in the same way that the Lagrangian integral scale is half of the FPIT for this model. The Eulerian integral scale might correspondingly be expected to be equal to $T_{\max}$ for model (b). In the contour plots, the vertical axis is therefore chosen to represent $T_{\max}/(2\tau_L)$ for model (a) and $T_{\max}/\tau_L$ for model (b). The horizontal axis represents $1/\beta$ for all model combinations.

In general, for small β (that is the large eddy lengths) τ_E approaches $T_{\max}/2$ for the constant $t_{\max}$ case. For larger values of β, however, $T_{\max}$ must exceed $2\tau_E$, due to the influence of the finite eddy length. In general, the value of β at which this influence is felt is smaller for the exponentially-distributed eddy length model (a)(ii) than for the constant-length model (a)(i). For the exponentially distributed $t_{\max}$, τ_E approaches the anticipated value of $T_{\max}$ as $\beta \to 0$. The influence of the finite eddy length is felt at smaller values of β for model (a)(i), compared with (b)(i). In general, larger values of $T_{\max}$ are required if the eddy length is distributed exponentially than for the constant eddy length case. A similar observation can be made on comparing the exponential and constant $t_{\max}$ models. Overall, the greatest influence of the finite eddy length is observed in model (b)(ii), with exponentially-distributed length and time scales. The eddy length has least influence for the constant eddy scales of model (a)(i).

Table 1. Eulerian integral scales for "standard models"

	(a)(i)	(a)(ii)	(b)(i)	(b)(ii)
β = 0.00	1.000	1.000	1.000	1.000
β = 0.25	1.000	0.940	0.989	0.857
β = 0.50	0.995	0.892	0.892	0.773
β = 1.00	0.937	0.820	0.817	0.671

The importance of $T_{\max}$ can be seen if we consider Eulerian integral scales given by the "standard" EIM obtained by setting $T_{\max}$ equal to 2 for model (a) and equal to 1 for model (b). The resulting values of τ_E for β = 1, 0.5 and 0.25 are given in table 1. It might naively be anticipated that each of these models would lead to an Eulerian integral scale equal to the Lagrangian integral scale. Clearly this is not the case unless the turbulence structure parameter β is small. However, many Lagrangian models have assumed that β = 1 and this leads to Eulerian integral time scales which are significantly less than the corresponding Lagrangian scales. For the exponentially-distributed length and time scales in model (b)(ii), τ_E is only 67% of τ_L! For the constant length and time scale model (a)(i), τ_E is 94% of τ_L. It is clearly possible that this property of the standard EIM could lead to difficulties in modelling the dispersion of high-inertia particles, especially if attempting to model a physical system in which $\tau_E > \tau_L$. It should be pointed out that specification of $T_{\max}$ enables us to *control* the ratio τ_E/τ_L, the ratio can also be forced to be less than 1 if necessary.

Table 2 shows the values of $T_{\max}$ necessary to give Eulerian integral scales of 0.5, 1 and 1.5 for values of β equal to 1, 0.5 and 0.25. It is again clear that model (a)(i) is the least-affected by the finite eddy length, with (b)(ii) being the most highly-influenced. For model (b)(ii), we note that the required value of $T_{\max}$ is always significantly greater than the resulting τ_E. Model (a)(i) is only influenced by the eddy length if either β is large (that is close to 1) or τ_E is much greater than τ_L (or both!). Table 2 confirms the claim that that models (a)(ii) and (b)(i) are intermediate between (a)(i) and (b)(ii).

4. NUMERICAL RESULTS

The numerical simulations compare the performance of the various EIM schemes with the analytical solution of Reeks (1977). Here we choose the case β = 0.359, $\tau_E/\tau_L = 1.4$. For this case, the required values of $T_{\max}/\tau_L$ are 2.80 [(a),(i)]; 3.16 [(a),(ii)]; 1.51 [(b),(i)]; 1.91 [(b),(ii)]. As noted above, when either $t_{\max}$ or the eddy length is distributed exponentially, the ratios $T_{\max}/\tau_E$ for this fairly small value of β are significantly greater than the anticipated ratios of 2.8 (constant $t_{\max}$) or 1.4 (exponential $t_{\max}$). In each simulation, 20,000 computational particles were used, each starting from rest at t = 0. In this case, particles take time to acquire turbulence. Particle turbulence was allowed to develop by ensuring sufficiently long computation times (Graham 1996c).

Figure 2 illustrates comparisons between numerical and analytical results for each model combination, for three different values (0.1, 1 and 10) of $g^* = g/(u'/\tau_L)$. The horizontal axis represents the Stokes number τ_p/τ_L and the vertical axis represents a dimensionless dispersion coefficient $D = D/(u'^2\tau_L)$.

Table 2. Values of $T_{\max}$ for various B and τ_E in all model combinations

		Model (i)			Model (ii)		
		τ_E=0.5	τ_E=1.0	τ_E=1.5	τ_E=0.5	τ_E=1.0	τ_E=1.5
	β = 0.25	1.000	2.000	3.002	1.000	1.034	2.135
Model (a)	β = 0.50	1.000	2.011	3.101	1.067	2.271	3.606
	β = 1.00	1.006	2.159	3.559	1.135	2.532	4.154
	β = 0.25	0.500	1.012	1.562	0.549	1.194	1.929
Model (b)	β = 0.50	0.506	1.084	1.788	0.597	1.374	2.314
β = 1.00	0.542	1.313	2.333	0.687	1.717	> 3	

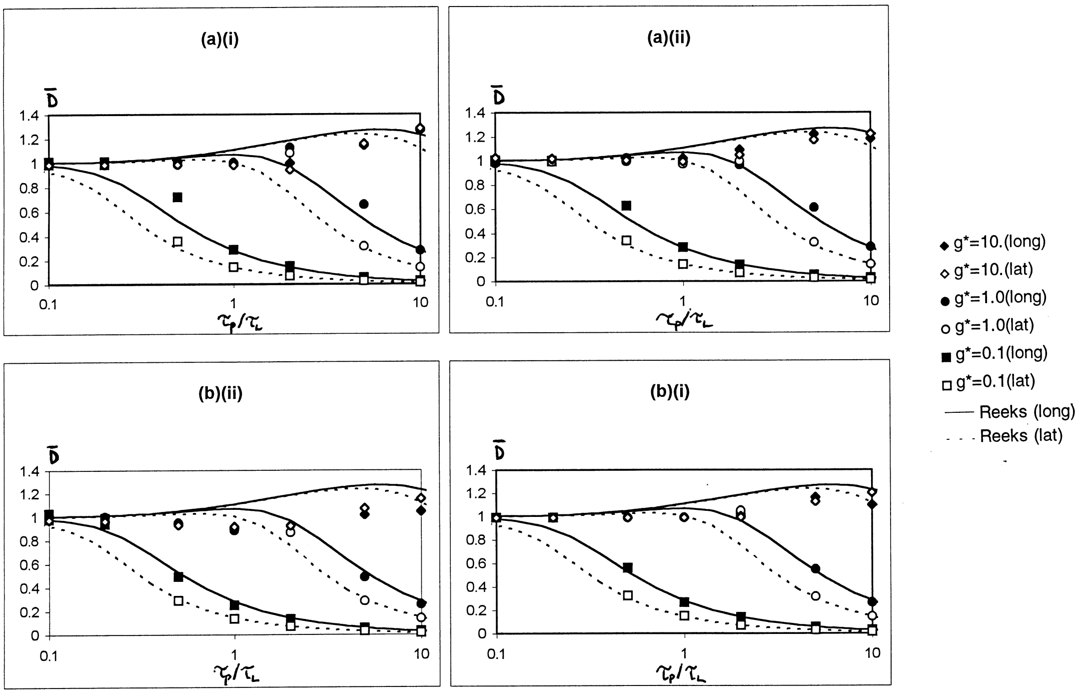

Figure 2. Particle dispersion coefficients vs. particle Stokes number for all model combinations.

For each model, the main effects are modelled reasonably well. Correct specification of the eddy length and FPIT distributions ensures that the CTE is well catered for with each model. The dispersion coefficient therefore decreases as the drift velocity $g\tau_p$ increases and the predictions of the model are well in line with the analytical solution. For small g, the increased dispersivity of high-inertia particles is predicted by all of the models, so that the inertia effect is also allowed for in each of the models, although perhaps not as well by model (b)(ii) as by the other models. Finally, in agreement with the analytical solution each model predicts that (i) for low drift velocities, particle dispersion coefficients are identical in all directions, and (ii) for high drift velocities, the "lateral" dispersion coefficient (that is at right angles to the drift velocity) is exactly half of the "longitudinal" coefficient (parallel to the drift). The continuity effect is therefore accounted for with each model. It is interesting to note that the method of halving interaction times in directions perpendicular to the drift velocity is equally as effective in modelling the continuity effect when the eddy scales are random as it is when the scales are constant.

In fact, the differences in computed dispersion coefficients between the different models are slight over the whole range of Stokes numbers. This perhaps indicates that it is the correct specification of the integral length and time scales (via l_e, t_e and $t_{\max}$) which is most important in determining the long-time dispersion characteristics, even for finite-inertia particles. Conversely, differences in correlation functions (or, equivalently, wave-number or frequency spectra) are less important. Short- and medium-time behaviour will be, of course, more susceptible to these differences.

There are, of course, some discrepancies between the numerical and analytical results. All of the models slightly under-predict the dispersion coefficients for Stokes numbers around 1 for the low-gravity case (and the predictions of model (b)(ii), in particular, show a *decrease* in dispersion coefficient in this region). All of the models appear to slightly over-predict dispersion for low-to-medium Stokes numbers (0.2–0.5) for the high-gravity case and for Stokes numbers around 1 for the intermediate-gravity case. It is unlikely that this overprediction can be related to the method used to determine the interaction times, described in Section 2. The threshold between the two different expressions used for the interaction time is at a drift velocity equal to the turbulence intensity, whereas this over-prediction appears to occur if the drift velocity is between two and five times the turbulence intensity.

Perfect agreement with the analytical solution should not be expected for any model because of the differences between the velocity auto-correlations in Reeks' solution and in the EIM. However, agreement in the limiting cases ($\tau_p \to 0$, $\tau_p \to \infty$, $g \to 0$ and $g \to \infty$) *should* be expected. The indications from the numerical simulations are that this exact agreement would be reached in the limiting cases. Overall, we conclude that the modified eddy interaction models with random time scales considered here successfully account for the CTE, inertia and continuity effects. Good levels of performance are demonstrated for a wide range of particle sizes and drift velocities. It is also worth noting that each modified EIM used in this paper performs significantly better than the standard EIM, which is capable of modelling the CTE but not the inertia and continuity effects.

Graham and James (1996) show that an eddy interaction model having a random fluid particle interaction time with mean T can lead to a stationary model of the turbulence only if $T/2 < \tau_L < T$ (where, as above τ_L is the Lagrangian integral scale of the model turbulence). For the exponentially-distributed time scales, $\tau_L = T$, whereas for the constant time scale model, $\tau_L = T/2$. Similar reasoning leads to the inequality $L/2 < \Lambda_E < L$, where L is the mean eddy length. Again, for the exponentially-distributed length scale, $\Lambda_E = L$ and $\Lambda_E = L/2$ for the constant eddy length model. The exponentially-distributed scale and the constant scale models therefore represent the extremes of the possible eddy interaction models. Because the method specified in Section 2 works for all combinations of these extreme cases, it can be inferred that it will work for *any* combination of eddy length and time scales, so long as the integral length and time scales are set properly. Of course, new $\tau_E(T_{\max}/\tau_L, \beta)$ relationships will be produced for combinations other than those given here.

In principle, however, this relationship can be determined as here by numerical integration. In common with any numerical model used to predict particle dispersion, the main problem is to have good estimates for the integral scales. Determination of length scales appears not to be problematical and methods based upon Taylor's "frozen turbulence" hypothesis can be used (Bradshaw 1971). It can be more difficult to measure integral time scales, although experimental methods capable of measuring τ_L and τ_E have recently become available (Beckel *et al.* 1995). Additionally, direct numerical simulations can be used to model simple flows (Squires and Eaton 1991) and *all* of the integral scales are available from the simulation results. In the meantime, values of the integral scales must be estimated from empirical correlations or as functions of the turbulence kinetic energy κ and its rate of dissipation ε in $\kappa - \varepsilon$ and other turbulence models (Graham and James 1996).

5. CONCLUSIONS

The performance of variants of the eddy interaction model of Gosman and Ioannides (1981) has been investigated. Each different model has a different combination of randomly-sampled eddy length and time scales. Each has been designed to account for the crossing trajectories, inertia and continuity effects. We note that

(i) the CTE is accounted for by specifying the correct forms of the eddy length and FPIT distributions;
(ii) by specifying the distribution of $T_{\max}$ (which depends on the model parameters), the inertia effect is catered for;
(iii) the continuity effect is modelled by allowing different interaction times in different coordinate directions.

Numerical results using different models have been compared with the analytical solution of Reeks (1977). The results from each model agree well with the analytical solution for a range of particle sizes and drift velocities. Constant eddy scales and exponentially-distributed scales represent the extremes of the possibilities available in eddy interaction models. The method has been shown to perform well for all possible combinations of these extremes. The method described in Section 2 can therefore be expected to work for any combination of length and time scale distributions, so long as these are chosen to give the correct integral length and time scales.

Acknowledgements—An early draft of this paper was presented at the 8th Workshop on Two-Phase Flow Prediction in Merseburg, Germany. Financial assistance given to the author by the Workshop organisers is gratefully acknowledged.

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